

U.G. 5th Semester Examination-2020**PHYSICS****[HONOURS]****Discipline Specific Elective (DSE)****Course Code : PHY-H-DSE-T-01****(Advanced Mathematical Physics-I)**

Full Marks : 40

Time : $2\frac{1}{2}$ Hours*The figures in the right-hand margin indicate marks.**Candidates are required to give their answers in their own words as far as practicable.***GROUP-A**

1. Answer any **five** questions : $2 \times 5 = 10$
- Find the value of $L[e^{ax}]$, where a is a constant and L represents the Laplace transformation.
 - Write any two properties required for an arbitrary set of n objects to form a linear vector V_n .
 - Prove that for every vector $|\varphi\rangle$ in V_n , there exists an inverse under addition, $|\neg\varphi\rangle$ such that $|\varphi\rangle + |\neg\varphi\rangle = |0\rangle$.
 - Write down the steps that are followed in the Gram-Schmidt orthogonalization method for

[Turn Over]

converting a linearly independent basis into an orthonormal one.

- State the first and second shifting theorems in connection to Laplace Transformation.
- In a Cartesian coordinate system, the distinction between the contravariant and the covariant tensors vanishes. Explain.
- What do you mean by a Tensor of zero order? Give an example.
- Write down the relationship between alternate and Kronecker tensor.

GROUP-BAnswer any **three** questions: $10 \times 3 = 30$

- Write down the fundamental form and the metric tensor of the Minkowski space in spherical coordinates. Why is Minkowski space not Euclidean?
 - Calculate the components of the Riemann-Christoffel tensor of the space defined by metric $ds^2 = (dx^1)^2 + (dx^2)^2 + (dx^3)^2 - e^{-t}(dt)^2$.
 - Write down the elaborate form of inertia tensor for a continuous body in Cartesians.
 - What are the diagonal and the off-diagonal elements stand for? $3+2+3+2$
- Show that the process of contraction of an Nth-

order tensor produces another tensor, of order $N - 2$.

- b) State and prove the 'quotient law' for tensors.
 c) Show that T_{ij} given by

$$T = [T_{ij}] = \begin{pmatrix} x_2^2 & -x_1x_2 \\ -x_1x_2 & x_1^2 \end{pmatrix}$$

are the components of a second-order tensor.
 3+5+2

4. a) Considering $A = [a_{ij}]$, $B = [b_{ij}]$, and that

$$B = A^{-1} \text{ show the relationship } \frac{\partial a}{\partial u^k} = ab^{ij} \frac{\partial a_{ij}}{\partial u^k}$$

by taking that the determinant $a = |A|$.

- b) If $f(x)$ is a periodic function of period $P > 0$, that is, if $f(x+P) = f(x)$, then show that

$$L[f(x)] = \frac{1}{1 - e^{-pP}} \int_0^P e^{-px} f(x) dx$$

5+5

5. a) State and prove the Laplace transform of the derivative of $f(t)$. Hence show that Laplace transform of derivatives of order n can be represented as

$$L[f^n(t)] = s^n L[f(t)] - s^{n-1} f(0) - s^{n-2} f'(0) - s^{n-3} f''(0) - \dots - f^{n-1}(0)$$

- b) Solve the initial value problem using Laplace transformations:

$$y'' + y = 0; y(0) = y'(0) = 0 \text{ and } f(t) = 0 \text{ for } t < 0 \text{ but } f(t) = 1 \text{ for } t \geq 0.$$

- c) Let $Y_z = AX_z$ be a linear transformation expressed concerning some original Z -basis $\{Z_1, Z_2, \dots, Z_n\}$. What is the expression for this same transformation when expressed w.r.t. some arbitrary other W -basis?
 4+3+3

6. a) What do you mean by linear mapping or linear transformation? Explain with a suitable example.

- b) When do the two linear spaces are said to be homomorphic and isomorphic?

- c) Prove that A linear transformation $Y = AX$ is nonsingular if and only if A , the matrix of the transformation, is nonsingular.

- d) Given the basis vectors A, B, C , below, use the Gram-Schmidt method to find an orthonormal set of basis vectors e_1, e_2, e_3 .

$$A = (0,0,5,0), B = (2,0,3,0) \text{ and } C = (7,1,-5,3)$$

3+2+2+3