U.G. 5th Semester Examination - 2020

MATHEMATICS

[PROGRAMME]

Skill Enhancement Course (SEC)

Course Code : MATH-G-SEC-T-3A&B

Full Marks : 40 Time : 2 Hours

The figures in the right-hand margin indicate marks. Candidates are required to give their answers in their own words as far as practicable.

Answer all the questions from selected Option.

OPTION-A

MATH-G-SEC-T-3A

1. Answer any **five** questions: 2×5=10

 a) If $f(-x)=f(x)$ for all real x, then prove that $\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx$.

 b) Evaluate $\int_0^{2\pi} |\sin x|\,dx$.

 c) If a function *f(x)* is periodic with period *T*, then prove that $\int_a^b f(x)\,dx = \int_{a+nT}^{b+nT} f(x)\,dx$, *n* is an integer.

 d) Evaluate $\int_0^1\int_0^2 x^3 y\,dxdy$.

 e) Find the length of the curve $y = \log \sec x$ from the interval $x = 0$ to $x=\frac{\pi}{3}$.

[Turn over]

 f) Evaluate $\lim_{n\to\infty}\left[\frac{1}{n+m}+\frac{1}{n+2m}+\cdots+\frac{1}{n+nm}\right]$.

 g) Obtain the reduction formula for $\int \sec^n x\,dx$.

 h) Evaluate $\int_0^1\int_0^\pi\int_0^\pi y\sin z\,dxdydz$.

2. Answer any **two** questions: 5×2=10

 a) Evaluate $\int_0^1\int_0^{2-x}\int_0^{2-x-y}\,dxdydz$.

 b) If $I_n=\int_0^{\frac{\pi}{4}}\tan^n x\,dx$ (*n* being an integer greater than 1), then prove that $\int_0^{\frac{\pi}{4}}\tan^8 x\,dx=\frac{\pi}{4}-\frac{76}{105}$.

 c) Find the area of the region bounded by the cardioide $r=2(1+\cos\theta)$.

 d) Find the volume of the tetrahedron bounded by the coordinate planes and the plane $x+y+z=1$.

3. Answer any **two** questions: 10×2=20

 a) Evaluate $(i)\int\frac{\sin x}{\sqrt{1+\sin x}}\,dx$, $(ii)\int\frac{x^2+2x+3}{\sqrt{1-x^2}}\,dx$. 5+5

 b) i) If $I_n=\int_0^{\frac{\pi}{2}} x^n\sin x\,dx$ (*n* is a positive integer), then prove that $I_n + n(n-1)I_{n-2} = n\left(\frac{\pi}{2}\right)^{n-1}$. 5

 ii) Find the volume of the solid obtained by the revolution of the cissoid $y^2(2a-x)=x^3\ (a>0)$ about its asymptote. 5

c) i) Find the length of the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$. 5

ii) Find the volume of the solid D, where D is the intersection of the solid sphere $x^2 + y^2 + z^2 \leq 9$ and the solid cylinder $x^2 + y^2 \leq 1$. 5

d) i) Find the area of the loop of the curve $a^3y^2 = yx^4(b + x)$ $(a > 0)$. 5

ii) Find the value of

$$\lim_{n\to\infty}\left\{\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right)\cdots\left(1+\frac{n}{n}\right)\right\}^{\frac{1}{n}}.$$

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OPTION-B

MATH-G-SEC-T-3B

(Vector Calculas)

1. Answer any **five** questions: 2×5=10

a) If $\vec{\alpha} = t^2\hat{i} - t\hat{j} + (2t-3)\hat{k}$ and $\vec{\beta} = (2t-3)\hat{i} + \hat{j} - t^2\hat{k}$, where $\hat{i}, \hat{j}, \hat{k}$ have their usual meaning, then find $\frac{d}{dt}\left(\vec{\alpha} \times \frac{d\vec{\beta}}{dt}\right)$ at $t = 1$.

b) Find $\vec{\nabla}\Phi$ with $\Phi = \frac{1}{\sqrt{x^2 + y^2 + z^2}}$.

c) Show that $\vec{F} = (y^2 \cos x + z^3)\hat{i} + (2y\sin x - 4)\hat{j} + (3xz^2 + 2)\hat{k}$ is a conservative force field.

d) If $\vec{F} = 3xy\hat{i} - 2x^2\hat{j}$, then evaluate $\int_C \vec{F}.\vec{r}$, where C is the curve $x = 2y^2$ on the xy- plane from the point (0, 0) to (2, 2).

e) Define irrotational and solenoidal vectors.

f) Evaluate $\int \vec{A} \times \frac{d^2\vec{A}}{dt^2}\, dt$.

g) If $\vec{A}$ has constant magnitude then show that $\vec{A} \times \frac{d\vec{A}}{dt} = 0$.

h) Find a unit normal to the surface $2x^2y + 3yz = 4$ at the point (1, –1, 2).

2. Answer any **two** questions: 5×2=10

a) If $\frac{d\vec{a}}{dt} = \vec{r} \times \vec{a}$ and $\frac{d\vec{b}}{dt} = \vec{r} \times \vec{b}$ then show that $\frac{d}{dt}(\vec{a} \times \vec{b}) = \vec{r} \times (\vec{a} \times \vec{b})$ where $\vec{r}$ is a constant vector and $\vec{a}, \vec{b}$ are vector functions of a scalar variable t .

b) If the vectors $\vec{A}$ and $\vec{B}$ be irrotational, then show that the vector $\vec{A} \times \vec{B}$ is solenoidal.

c) Evaluate $\int_C \vec{A}\cdot d\vec{r}$ from (0, 0, 0) to (1, 1, 1) along the path $C: x=t, y=t^2, z=t^3$ where $\vec{A}=(3x^2+6y)\hat{i}-14yz\hat{j}+20xz^2\hat{k}$

d) Prove that $\nabla^2\left(\frac{1}{r}\right)=0$ where $r=\sqrt{x^2+y^2+z^2}$.

3. Answer any **two** questions: 10×2=20

a) i) Find constants *a, b* and *c* so that $\vec{V}$ is irrotational where

$$\vec{V}=(-4x-3y+az)\hat{i}+(bx+3y+5z)\hat{j}+(4x+cy+3z)\hat{k}\cdot$$

ii) Show that $\vec{V}$ can be expressed as the gradient of a scalar function. 5+5

b) Evaluate $\iint_S \vec{A}\cdot\vec{n}dS$ where $\vec{A}=18z\hat{i}-12\hat{j}+3y\hat{k}$ and S is the part of the plane $2x+3y+6z=12$ which is located in the first octant. 10

c) i) Let V be the closed region bounded by the surfaces $x=0, x=2, y=0, y=6, z=x^2$, $z=4$ and $\vec{F}=y\hat{i}+2x\hat{j}-z\hat{k}$. Find $\iiint_V \vec{\nabla}\times\vec{F}dV$.

ii) Find a unit normal to the surface $2x^2y+3yz=4$ at the point (1, –1, 2).

6+4

d) i) If $\vec{F}$ is a conservative field then prove that curl $\vec{F}=\vec{0}$.

ii) Conversely, if curl $\vec{F}=\vec{0}$ then prove that $\vec{F}$ is a conservative field. 5+5

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