U.G. 5th Semester Examination - 2020

MATHEMATICS

[HONOURS]

Discipline Specific Elective (DSE)

Course Code : MATH-H-DSE-T-01A

(Linear Programming)

Full Marks : 60 Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks. Candidates are required to give their answers in their own words as far as practicable.

1. Answer any **ten** questions: 2×10=20

i) Check for convexity of the set $X = \{(x_1, x_2) \in \mathbb{R}^2 : 9x_1^2 + 4x_2^2 \leq 36\}$.

ii) Find a basic feasible solution of the system of linear equations:

$$x_1 + 2x_3 = 1, x_2 + x_3 = 4.$$

iii) Define degenerate solution of an LPP.

iv) Write in standard form of the following LPP:

Maximize $Z = -x_1 + 2x_2$;

subject to

$2x_1 - 3x_2 \geq -6$;
$-x_1 + 8x_2 \leq -4$;
$x_1, x_2 \geq 0$.

[Turn over]

v) Write down the dual form of the following LPP:

Minimize $Z = -6x_1 - 2x_2 + 10x_3$;

subject to

$x_1 + x_2 - x_3 \geq 2$;
$7x_1 - 2x_3 \leq 3$,
$x_1, x_2, x_3 \geq 0$.

vi) Using graphical method solve the following LPP:

Maximize $Z = 3x_1 + x_2$;

subject to $2x_1 + 3x_2 \leq 6$;
$x_1 + x_2 \geq 1$;
$x_1, x_2 \geq 0$.

vii) Determine extreme point(s), if any, of the set $S = \{(x, y) : |x| \leq 3; |y| \leq 3\}$.

viii) Write in cannonical form of the following LPP:

Minimize $Z = 3x_1 + x_2$;

subject to $-2x_1 + 4x_2 \leq 3$;
$x_1 + 8x_2 \geq 5$;
$x_1, x_2 \geq 0$.

ix) Check for linear dependence of the vectors $\{(1, 2, 0), (0, 3, 1), (-1, 0, 1)\}$.

x) Solve the game with the pay off matrix $\begin{pmatrix} 6 & -3 \\ -3 & 0 \end{pmatrix}$.

xi) Using VAM, find a solution of the following transportation problem:

	P	Q	R	
A	5	1	8	10
B	7	4	5	25
C	6	8	4	5
	15	17	8	

xii) Show that whatever may be the values of a, the game with the following pay off matrix is strictly determinable:

	BI	BII
AI	2	6
AII	-4	a

xiii) Determine saddle point of the game with the pay off matrix

$$\begin{pmatrix} -5 & 3 & 1 & 20 \\ 5 & 5 & 4 & 6 \\ -4 & -2 & 0 & -5 \end{pmatrix}$$

xiv) Find the range and value of p and q to make the payoff element a_{22} a saddle point for the game whose payoff matrix (a_{ij}) is given by

	B1	B2	B3
A1	2	4	7
A2	10	7	q
A3	4	p	8

xv) How a maximization type of assignment problem can be converted to minimization type for solving through Hungarian method?

2. Answer any **four** questions: 5×4=20

a) Reduce the feasible solution $x_1 = 2, x_2 = 1$, $x_3 = 1$ of the system of linear equations, $x_1 + 4x_2 - x_3 = 5,\ \ 2x_1 + 3x_2 + x_3 = 8$ to a basic feasible solution.

b) Define a convex set. Show that the set of all feasible solutions of an LPP is a convex set.

c) Show that the LPP:

Minimize $Z = 3x_1 - 2x_2$;

subject to $x_1 - x_2 \le 1$;

$3x_1 - 2x_2 \le 6$;

$x_1, x_2 \ge 0$,

has unbounded solution.

d) For a transportation problem with m number of origins and n number of destinations, prove that the total number of basic variables does not exceed m+n–1.

e) Define extreme point of a convex set in E^n. Find the extreme points of the possible solutions of the LPP

Maximize $Z = 4x_1 + 7x_2$;

subject to $2x_1 + 5x_2 \leq 40$;

$x_1 + x_2 \leq 11$;

$x_1, x_2 \geq 0$.

f) Using graphical method, solve the following game with the following pay off matrix:

	B1	B2	B3	B4
A1	1	2	-3	7
A2	2	5	4	-6

3. Answer any **two** questions: 10×2=20

a) i) Solve the following LPP:

Maximize $Z = 2x_1 - 3x_2$;

subject to $-x_1 + x_2 \geq -2$;

$5x_1 + 4x_2 \leq 46$;

$7x_1 + 2x_2 \geq 32$;

$x_1, x_2 \geq 0$.

ii) In a factory, there are five machines which are to be assigned to five operators.

Operator I cannot operate Machine C and Operator III cannot operate Machine D. Find the optimal assignment schedule from the following assignment costs:

	A	B	C	D	E
I	5	5	-	2	6
II	7	4	2	3	4
III	9	3	5	-	3
IV	7	2	6	7	2
V	6	5	7	9	1

5+5

b) i) Find optimal solution of the following LPP by solving its dual:

Minimize $Z = 4x_1 + 3x_2 + 6x_3$;

subject to $x_1 + 3x_3 \geq 2$;

$x_2 + x_3 \geq 5$;

$x_1, x_2, x_3 \geq 0$.

ii) If a fixed number p is added with each element of a pay off matrix of game, then prove that the optimal strategies remain unchaged, while value of the game is increased by p.

Diagonalize the matrix

$$A = \begin{pmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{pmatrix}.$$

6+4

c) i) Solve the following transportation problem.

	D_1	D_2	D_3	D_4	
O_1	10	20	5	7	10
O_2	11	9	12	8	20
O_3	4	16	7	9	30
O_4	14	7	1	0	40
O_5	3	12	5	19	50
	60	60	20	10	

ii) A company is making two products, A and B. The cost of producing one unit of product A and B is Rs.70/- and Rs.100/-, respectively. As per the agreement, the company has to supply at least 300 units of product B to its regular customer. One unit of product A requires one machine-hour whereas product B has machine-hours available abundantly within the company. Total machine hours available for product A are 500 hours. One unit of each product A and B requires one labour-hour each and total of 800 labour-hours are available. The company wants to minimize the cost of production by satisfying the given requirements. Formulate the problem as a linear programming problem. 6+4

d) i) Solve the following LPP by two phase method:

Minimize $Z = 3x_1 + 5x_2$;

subject to $x_1 + 2x_2 \geq 8$;

$3x_1 + 2x_2 \geq 12$;

$5x_1 + 6x_2 \leq 60$;

$x_1, x_2 \geq 0$.

ii) Obtain dual of the following LPP:

Minimize $Z = x_1 - 6x_3$;

subject to $-x_1 + 3x_2 - 2x_3 \leq 12$;

$2x_2 - x_3 \geq 15$;

$-4x_1 + 3x_2 = 10$;

$x_1, x_2 \geq 0$

and x_3 is unrestricted in sign. 6+4

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