U.G. 5th Semester Examination - 2020

MATHEMATICS

[HONOURS]

Discipline Specific Elective (DSE)

Course Code : MATH-H-DSE-T-01

(Point Set Topology)

Full Marks : 60 Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks. Candidates are required to give their answers in their own words as far as practicable.

1. Answer any **ten** questions: 2×10=20

i) State Zorns Lemma.

ii) Consider R^2 with the topology having a subbase consisting of the collection of all straight lines. Identify the topology?

iii) Show that closures of any two disjoint open sets are disjoint.

iv) Show by an example that derived set of any arbitrary set is not necessarily closed.

v) Every closed function from a topological space onto other is open -Justify.

vi) Show that the characteristic function χ_A of a subset A of a topological space X is continuous on X if A is clopen in X.

vii) Show that every real valued continuous function f on a compact space (X, τ) attains its least and greatest values.

viii) State Alexandroff's subbase theorem.

ix) $R^n \backslash \{1\}$ (where n is a natural number > 1) is connected. Justify.

x) Give an example to show that continuous image of a locally connected space need not be locally connected.

xi) Show that each component of a topological space is closed.

xii) Give an example of a noncompact locally compact space.

xiii) A quotient map is always open, justify.

xiv) What can you say about the nature of a convergent sequence in an uncountably infinite set with cocountable topology?

xv) Show that the components of open subsets of the real line are open intervals.

2. Answer any **four** questions : 5×4=20

i) Let A be a subset of a topological space (X, τ) and $x \in X$. Prove that $x \in cl(A)$ if and only if every basic open neighbourhood of x intersects A.

ii) Show that a map $f:(X,\tau) \to (Y,\tau^1)$ is continuous if and only if $f:(\bar{A}) \subseteq \overline{f(A)}$, for any subset A of X, where A denotes the closure of A.

iii) Show that a space (X, τ) is connected if and only if no continuous function on X into the discrete two point space {0, 1} is surjective.

iv) Show that a topological space (X, τ) is compact if and only if for every collection of closed sets $\{F_\alpha : \alpha \in A\}$ in X possessing FIP, the intersection $\cap\{F_\alpha : \alpha \in A\}$ of the entire collection is nonempty.

v) Let (X, τ) denote the topological product of the family of compact spaces $\{(X_\alpha, \tau_\alpha) : \alpha \in \wedge\}$. Show that (X, τ) is compact. Is the converse true?

vi) Let *f* be a continuous function from a compact metric space (X, d_X) into a metric space (Y, d_Y). Show that *f* is uniformly continuous.

3. Answer any **two** questions : 10×2=20

i) a) Let (Y, τ_Y) be a subspace of a topological space. Prove that a set F is closed in (Y, τ_Y) if and only if $F = Y \cap K$, for some set K closed in (X, τ).

b) Consider the set R of reals. Let $\tau = \{A \subseteq R : 0 \in A\} \cup \{\phi\}$. Then (R, τ) is a topological space. Find $\overline{\{O\}}$. Is this topology compact? Is this topology connected?

ii) a) Show that every countable compact metric space is totally bounded.

b) Show that every path connected space is connected. Is the converse true?

iii) a) Let f : X → Y be a continuous function from a space X onto a space Y. Show that if A is dense in X, then its image f(A) is dense in X.

b) Show that a continuous real valued function on a connected space X assumes all values between any two given values.

iv) a) Show that $f : X \to Y$, where $f(X) = Y$, is a quotient mapping if and only if for each closed $F \subseteq Y$ the following conditions are equivalent:

1) F is closed in Y;

2) $f^{-1}(F)$ is closed in X.

b) Let $\{(X_\alpha, \tau_\alpha) : \alpha \in \wedge\}$ be a given family of topological spaces and X, τ) their topological product. Prove that each (X_α, τ_α) can be embedded in the product space (X, τ).