

U.G. 3rd Semester Examination - 2019

PHYSICS

[HONOURS]

Course Code : PHYS(H)CC-05-T



Full Marks : 40

Time : 2½ Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP-A

1. Answer any five questions: 2×5=10

a) State Dirichlet's conditions for a Fourier series.

b) What do you mean by the orthogonality special functions?

c) Evaluate $\Gamma\left(-\frac{1}{2}\right)$.

d) Prove the following property of the Beta function $\beta(l, m) = \beta(m, l)$.

e) What are the singular points of a second order linear differential equations?

f) What do you mean by random error?

[Turn over]

- g) Write down the Parseval's formula.
- b) Write down the relation between the Beta and Gamma function.

GROUP-B

2. Answer any two questions: 5×2=10

a) i) When does a Laguerre function transform to a Laguerre Polynomials? 2

ii) Find the constant a_0 of the Fourier series for the function $f(x)=x$ in $0 \leq x \leq 2\pi$. 3

b) i) Evaluate $\int_0^{\infty} \sqrt{x} e^{-x} dx$. 2

ii) Find the regular singular points of the differential equation

$$2x^2 \frac{d^2y}{dx^2} + 3x \frac{dy}{dx} + (x^2 - 4)y = 0 \quad 3$$

c) Prove the orthogonality condition of Legendre Polynomials

$$\int_{-1}^1 P_m(x) P_n(x) dx = 0, \quad m \neq n \quad 5$$

- d) Write down the Bessel's differential equation of n-th order and its solution $J_n(x)$. 5

GROUP-C

3. Answer any two questions: 10×2=20

a) i) Find the Fourier series for the function $f(x) = e^{ax}$ for $0 < x < \pi$, where a is constant. 5

ii) Find the integral $\int_0^{\frac{\pi}{2}} \sin^p \theta \cos^q \theta d\theta$ using

$\beta(m, n)$ function in terms of $\Gamma(x)$ function. 5

b) Using Froberius method, obtain a series solution in powers of x for differential equation:

$$2x(1-x) \frac{d^2y}{dx^2} + (1-x) \frac{dy}{dx} + 3y = 0 \quad \text{about } x=0.$$

10

c) i) Find three dimensional Laplace's equation in cylindrical co-ordinates. 6

ii) Prove that

$$\int_{-1}^{+1} P_n(x)(1-2xt+t^2)^{\frac{1}{2}} dx = \frac{2t^n}{2n+1}, \text{ given,}$$

$$\int_{-1}^{+1} [P_n(x)]^2 dx = \frac{2}{2n+1} \text{ where } n \text{ is a}$$

positive integer. 4.

d) A tightly stretched string with fixed end points at $x=0$ and $x=l$ is initially in a position given by

$$y = y_0 \sin^3\left(\frac{\pi x}{l}\right).$$

If it is released from rest from position x (within $0 < x < l$), find the displacement $y(x, t)$. 10

