U.G. 2nd Semester Examination - 2020

MATHEMATICS

[GENERIC ELECTIVE]

Course Code : MTMH-GE-T-02

Full Marks : 60 Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

The symbols and notations have their usual meanings.

1. Answer any **ten** questions: 2×10=20

i) Find the differential equation of all parabolas having their axes parallel to y-axis.

ii) Verify that $(x^2+y^2+2x)dx+2ydy=0$ becomes an exact differential equation when both sides of it are multiplied by e^x.

iii) Solve: $\frac{dy}{dx}+1=e^{x-y}$.

iv) Examine whether the following differential equation is an exact differential equation, $ydx+(x+\cos y)dy=0$.

v) Eliminate the parameters a and b from the following primitives: $xy=ae^x+be^{-x}$.

vi) Find the integrating factor of the differential equation $xcos\,x\frac{dy}{dx}+y(xsin\,x+\cos x)=1$.

vii) Find the order and degree of the differential equation $y(\frac{dy}{dx})^2+2x\frac{dy}{dx}-y=0$.

viii) Solve: $2xydx-(x^2-y^2)dy=0$.

ix) Find the partial differential equation by eliminating the arbitrary constants from the relation: $z=ax+(1-a)y+b$.

x) Find the partial differential equation by eliminating the arbitrary functions from the relation: $z=\alpha(x+iy)+\beta(x-iy)$.

xi) Find Charpit's auxiliary equations for the following partial differential equation: $z=px+qy+p^2+q^2$.

xii) Solve: $\frac{dx}{z}=\frac{dy}{0}=\frac{dz}{-x}$.

2. Answer any **four** questions: 5×4=20

i) Solve the partial differential equation: $x^2p+y^2q=(x+y)z$.

ii) Solve by Charpit's method: $p^2x+q^2y=z$.

iii) Solve by the method of variation of parameter: $(D^2-2D-3)y=64xe^{-x}$.

iv) Solve: $x^3\frac{d^3y}{dx^3} - x^2\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} - 2y = x^3$.

v) Solve the following system of simultaneous equations: $\frac{dy}{dx} + 2y - 3z = x$, $\frac{dz}{dx} + 2z - 3y = e^{2x}$.

vi) Obtain the complete primitive and singular solution of the equation: $y = px + \sqrt{1+p^2}$.

3. Answer any **two** questions: 10×2=20

i) a) Find the complete integral of the equation $p^3 + q^3 = 3pqz$.

b) Solve by Lagrange's method:
$z(x+y)\frac{\partial z}{\partial x} + z(x-y)\frac{\partial z}{\partial y} = x^2 + y^2$. 5+5

ii) a) Find the singular solution of $y = px + \frac{a}{p}$.

b) Solve: $(yz + z^2)dx - xzdy + xydz = 0$.

5+5

iii) a) Solve $(D^2 - 6D + 9)y = x^3e^{3x}$, assuming that e^{3x} is a solution of its reduced equation.

b) Find the integral surface of the linear equation
$x(y^2 + z)p - y(x^2 + z)q = (x^2 - y^{2)}z$
containing the straight line $x + y = 0, z = 1$.

5+5

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